Day 8	**Numbering Systems**	**29-11-2015** **30 -11-2015**

Numbering Systems

- **Numbering Systems** has four types:

1-	Binary (**0** , **1**)	has base is **2**	ثنائي
2-	Octal	has base **8**	ثماني
3-	Decimal	has base **10**	عشري
4-	Hexadecimal	has base **16**	سادس عشر

- Computer operates with electrical pulses:

  1 for **ON**

  0 for **OFF**

- **Examples to understand bases of numbering systems:**

Decimal (base **10**)

The decimal number consists of **ten** digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

e.g., $(\mathbf{8910})_{10} = \mathbf{8} \times \mathbf{1000} + \mathbf{9} \times \mathbf{100} + \mathbf{1} \times \mathbf{10} + \mathbf{0} \times \mathbf{0}$

$= \mathbf{8} \times \mathbf{10^3} + \mathbf{9} \times \mathbf{10^2} + \mathbf{1} \times \mathbf{10^1} + \mathbf{0} \times \mathbf{0^0}$

Biary (base **2**)

The binary number consists of **two** digits 0, 1

e.g., $(\mathbf{1010})_2 = \mathbf{1} \times \mathbf{2^3} + \mathbf{0} \times \mathbf{2^2} + \mathbf{1} \times \mathbf{2^1} + \mathbf{0} \times \mathbf{2^0}$

Octal (base **8**)

The octal number consists of **eight** digits 0, 1, 2, 3, 4, 5, 6, 7

e.g., $(\mathbf{7426})_8 = \mathbf{7} \times \mathbf{8^3} + \mathbf{4} \times \mathbf{8^2} + \mathbf{2} \times \mathbf{8^1} + \mathbf{6} \times \mathbf{8^0}$

Hexa (base **16**)

The hexa number consists of **sixteen** digits are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

[Where A=**10**, B=**11**, C=**12**, D=**13**, E=**14**, F=**15**]

e.g., $(\mathbf{2BC3})_{16} = \mathbf{2} \times \mathbf{16^3} + \mathbf{B} \times \mathbf{16^2} + \mathbf{C} \times \mathbf{16^1} + \mathbf{3} \times \mathbf{16^0}$

- **Converting Between Numbering Systems:** four conversion types

  1- any base to decimal

  2- decimal to any base

  3- octal/hexa to binary

  4- binary to octal/hexa

{Note that the base of each numbering system is written as subscript at the right of the number}

Octal number **4637** *is written as* $(\mathbf{4637})_8$

1- **any base** to **decimal**

Binary to Decimal

$$\begin{aligned} \text{e.g., } (10101)_2 &= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 16 + 0 + 4 + 0 + 1 \\ &= (21)_{10} \end{aligned}$$

Octal to Decimal

$$\begin{aligned} \text{e.g., } (472)_8 &= 4 \times 8^2 + 7 \times 8^1 + 2 \times 8^0 \\ &= 256 + 56 + 2 \\ &= (314)_{10} \end{aligned}$$

Hexa to Decimal

$$\begin{aligned} \text{e.g., } (2BC1)_{16} &= 2 \times 16^3 + B \times 16^2 + C \times 16^1 + 1 \times 16^0 \\ &= 8192 + 2816 + 192 + 1 \\ &= (11201)_{10} \end{aligned}$$

2- decimal to any base

Decimal to Binary

e.g., **$(364)_{10}$** **364 is placed between 256 and 512**

$$\begin{array}{r} 1 \\ 256\overline{)364} \\ \underline{256} \\ 108 \end{array}$$ >>>>> 1

$$\begin{array}{r} 0 \\ 128\overline{)108} \\ \underline{000} \\ 108 \end{array}$$ >>>>> 0

$$\begin{array}{r} 1 \\ 64\overline{)108} \\ \underline{64} \\ 44 \end{array}$$ >>>>> 1

$$\begin{array}{r} 1 \\ 32\overline{)44} \\ \underline{32} \\ 12 \end{array}$$ >>>>> 1

$$\begin{array}{r} 0 \\ 16\overline{)12} \\ \underline{00} \\ 12 \end{array}$$ >>>>> 0

$$\begin{array}{r} 1 \\ 8\overline{)12} \\ \underline{8} \\ 4 \end{array}$$ >>>>> 1

$$\begin{array}{r} 1 \\ 4\overline{)4} \\ \underline{4} \\ 0 \end{array}$$ >>>>> 1

$$\begin{array}{r} 0 \\ 2\overline{)0} \\ \underline{0} \\ 0 \end{array}$$ >>>>> 0

$$\begin{array}{r} 0 \\ 1\overline{)0} \\ \underline{0} \\ 0 \end{array}$$ >>>>> 0

364

2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128
2^8	256
2^9	**512**

$(364)_{10} = (101101100)_2$

Check **$(101101100)_2 = 1\ x2^8 + 0\ x2^7 + 1x2^6 + 1\ x2^5 + 0x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 0x2^0$**

$= 256 + 0 \quad + 64 \quad + 32 \quad + 0 \quad + 8 \quad + 4 \quad + 0 \quad + 0 \ = (364)_{10}$

Decimal to Octal

e.g., **$(687)_{10}$ the decimal number 687 is placed between 512 and 4096**

$$\begin{array}{r} 1 \\ 512\overline{)687} \\ \underline{512} \\ 175 \end{array}$$

>>>>> 1

$$\begin{array}{r} 2 \\ 64\overline{)175} \\ \underline{128} \\ 47 \end{array}$$

>>>>> 2

$$\begin{array}{r} 5 \\ 8\overline{)47} \\ \underline{40} \\ 7 \end{array}$$

>>>>> 5

$$\begin{array}{r} 7 \\ 1\overline{)7} \\ \underline{7} \\ 0 \end{array}$$

>>>>> 7

687 →

8^0	1
8^1	8
8^2	64
8^3	512
8^4	4096

$(687)_{10} = (1257)_8$

Decimal to Hexa

e.g., **687_{10} the decimal number 687 is placed between 256 and 4096**

$$\begin{array}{r} 2 \\ 256\overline{)687} \\ \underline{512} \\ 175 \end{array}$$

$>>>>>$ 2

$$\begin{array}{r} 10 \\ 16\overline{)175} \\ \underline{160} \\ 15 \end{array}$$

$>>>>>$ 10 = A

$$\begin{array}{r} 15 \\ 1\overline{)15} \\ \underline{15} \\ 00 \end{array}$$

$>>>>>$ 15 = F

687 →

16^0	1
16^1	16
16^2	256
16^3	4096

$$(687)_{10} = (2AF)_{16}$$

$$(687)_{10} = (1257)_{8} = (2AF)_{16}$$

3- **octal/hexa to binary**

Octal to Binary

Each number in octal code is represented by three binary digits

2^2	2^1	2^0
4	2	1

e.g., $(\mathbf{263})_8 = \mathbf{2} + \mathbf{6} + \mathbf{3}$

↓ ↓ ↓

$= 010 + 110 + 011$

$= (010110011)_2$

Hexa to Binary

Each number in hexa code is represented by four binary digits

2^3	2^2	2^1	2^0
8	4	2	1

A = 10

E = 14

e.g., $(\mathbf{2A4E})_{16} = \mathbf{2} + \mathbf{A} + \mathbf{4} + \mathbf{E}$

↓ ↓ ↓ ↓

$= 0010 + 1010 + 0100 + 1110$

$= (0010101001001110)_2$

4- binary to octal/hexa

Binary to Octal

Each three binary digits in binary code is represented by one octal number

2^2	2^1	2^0
4	2	1

e.g.,
$$\begin{array}{lccc} (1010)_2 = & 001 & + & 010 \\ & \downarrow & & \downarrow \\ = & 1 & + & 2 \\ = & (12)_8 & & \end{array}$$

e.g.,
$$\begin{array}{lccccc} (101111010)_2 = & 101 & + & 111 & + & 010 \\ & \downarrow & & \downarrow & & \downarrow \\ = & 5 & + & 7 & + & 2 \\ = & (572)_8 & & & & \end{array}$$

Binary to Hexa

Each four binary digits in binary code is represented by one hexa number

2^3	2^2	2^1	2^0
8	4	2	1

e.g., $(101001)_2 = 0010 + 1001$

$\downarrow \qquad \downarrow$

$= 2 + 9$

$= (29)_{16}$

e.g., $(1011000011)_2 = 0010 + 1100 + 0011$

$\downarrow \qquad \downarrow \qquad \downarrow$

$= 2 + C + 3$

$= (2C3)_{16}$

- **Adding Binary Numbers**

Note that:

$$\begin{array}{r} 0 \\ +0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ +0 \\ \hline 1 \end{array} \qquad \begin{array}{r} 0 \\ +1 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ +1 \\ \hline 10 \end{array}$$

e.g.,

$$\begin{array}{crrrr} & 1 & 1 & 1 & \\ & 0 & 1 & 0 & 1 \\ + & 0 & 1 & 1 & 1 \\ \hline & 1 & 1 & 0 & 0 \end{array}$$

$$\begin{array}{rrrrr} & & 1 & 1 & \\ 1 & 1 & 1 & 1 & \\ & 1 & 0 & 1 & 1 \\ + & 0 & 1 & 0 & 1 \\ + & 0 & 0 & 1 & 1 \\ + & 1 & 0 & 0 & 1 \\ \hline 1 & 1 & 1 & 0 & 0 \end{array}$$

• **Multiplying Binary Numbers**

Note that:

$$\begin{array}{r} 0 \\ x\ \ 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ x\ \ 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 0 \\ x\ 1 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ x\ 1 \\ \hline 1 \end{array}$$

e.g.,

$$\begin{array}{r} 101 \\ x\quad 11 \\ \hline 101 \\ 101 \\ \hline 1111 \end{array}$$

$$\begin{array}{r} 100111 \\ x\ \ 110101 \\ \hline 100111 \\ 000000 \\ 100111 \\ 000000 \\ 100111 \\ 100111 \\ \hline 100000010011 \end{array}$$